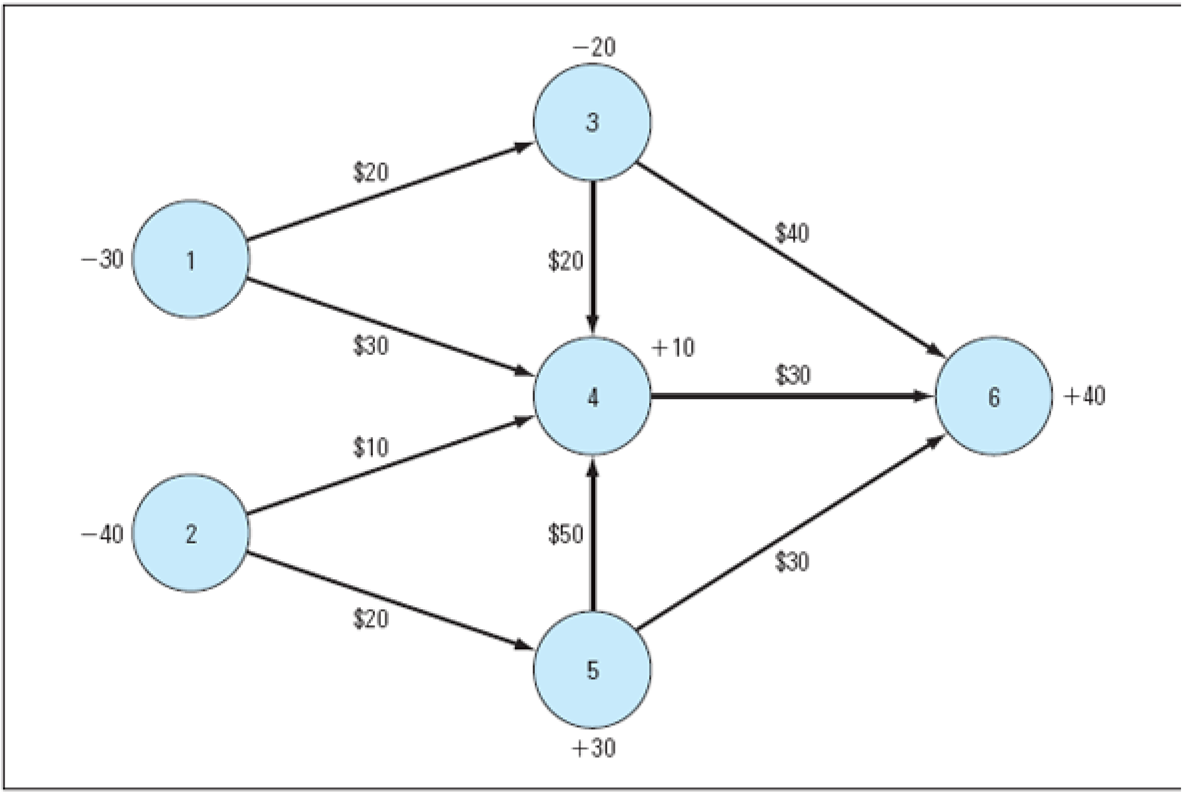
Network Modeling

1. A furniture manufacturer has warehouses in cities represented by nodes 1, 2, and 3 in Figure. The values on the arcs indicate the per unit shipping costs required to transport living room suites between the various cities. The supply of living room suites at each warehouse is indicated by the negative number next to nodes 1, 2, and 3. The demand for living room suites is indicated by the positive number next to the remaining nodes.



a. Identify the supply, demand, and transshipment nodes in this problem.
b. Use Solver to determine the least costly shipping plan for this problem.

Solution:

a. Supply nodes: 1, 2

Demand node: 6

Transshipment nodes: 3, 4 & 5

b. See

Furniture Distribution

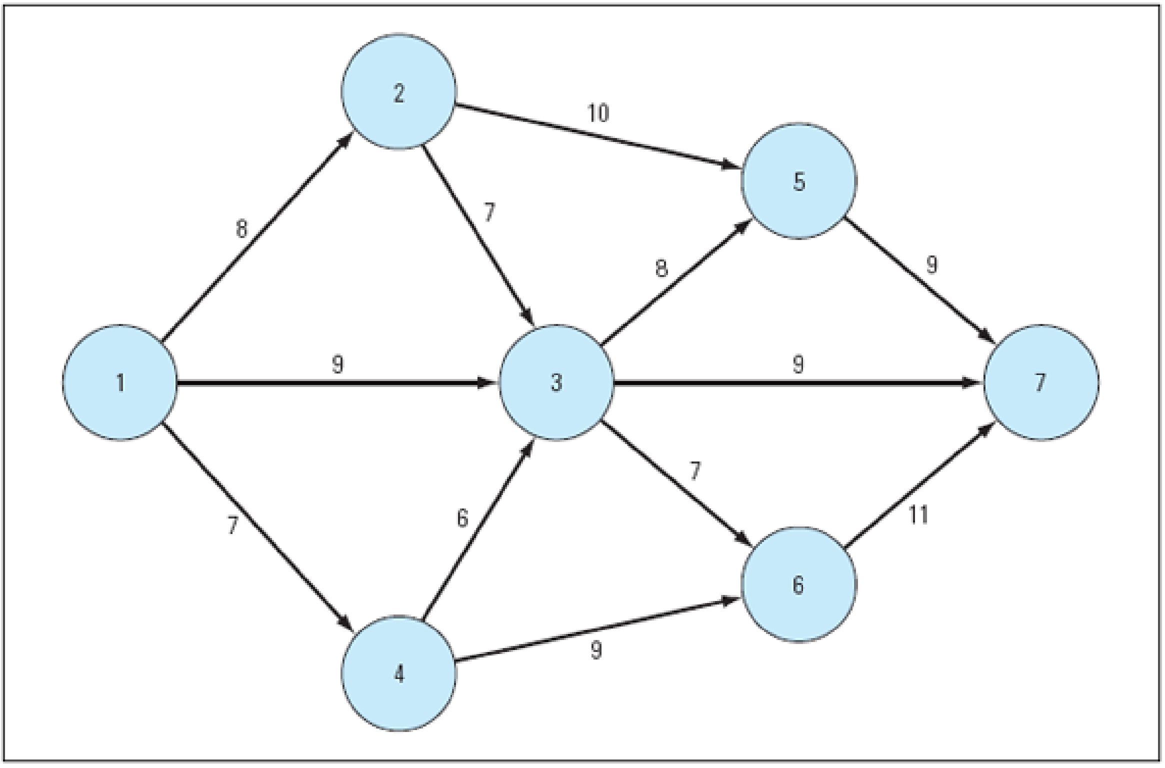
Ship:	Arcs From:	To:	Cost
20	1	3	$20.0
0	1	4	$30.0
10	2	4	$10.0
30	2	5	$20.0
0	3	4	$20.0
40	3	6	$40.0
0	4	6	$30.0
0	5	4	$50.0
0	5	6	$30.0
		Total:	$2,700.0

Node	Net Flow	Supply or Demand
1	-20	-30
2	-40	-40
3	-20	-20
4	10	10
5	30	30
6	40	40

Minimize: D14
By changing: A5:A13
Subject to: G5:G10>=H5:H10
A5:A13>=0

The solution is: X_{13}=20, X_{24}=10, X_{25}=30, X_{36}=40, with a minimum total cost of $2,700.

2. The graph in Figure represents various flows that can occur through a sewage treatment plant with the numbers on the arcs representing the maximum flow (in tons of sewage per hour) that can be accommodated. Formulate an LP model to determine the maximum tons of sewage per hour that can be processed by this plant.



Network flow model for the sewage treatment plant

Solution:

MAX X_{71}

ST $+X_{71} - X_{12} - X_{13} - X_{14} = 0$

$+X_{12} - X_{23} - X_{25} = 0$

$+X_{13} + X_{23} + X_{43} - X_{35} - X_{36} - X_{37} = 0$

$+X_{14} - X_{43} - X_{46} = 0$

$+X_{25} + X_{35} - X_{57} = 0$

$+X_{36} + X_{46} - X_{67} = 0$

$+X_{37} + X_{57} + X_{67} = 0$

$0 \leq X_{12} \leq 8$

$0 \leq X_{13} \leq 9$

$0 \leq X_{14} \leq 7$

$0 \leq X_{23} \leq 7$

$0 \leq X_{25} \leq 10$

$0 \leq X_{35} \leq 8$

$0 \leq X_{36} \leq 7$

$0 \leq X_{37} \leq 9$

$0 \leq X_{43} \leq 6$

$0 \leq X_{46} \leq 9$

$0 \leq X_{57} \leq 9$

$0 \leq X_{67} \leq 11$

Sewage Treatment Plant

Units of Flow	-- Arcs -- From	To	Upper Bound
8.0	1	2	8
9.0	1	3	9
7.0	1	4	7
0.0	2	3	7
8.0	2	5	10
1.0	3	5	8
0.0	3	6	7
9.0	3	7	9
1.0	4	3	6
6.0	4	6	9
9.0	5	7	9
6.0	6	7	11
24.0	7	1	1E+06
24.0	= Maximal Flow		

Node	Net Flow
1	0
2	0
3	0
4	0
5	0
6	0
7	0

Maximize: B18
By changing: B5:B17
Subject to: H5:H11=0
B5:B17<=E5:E17
B5:B17>=0

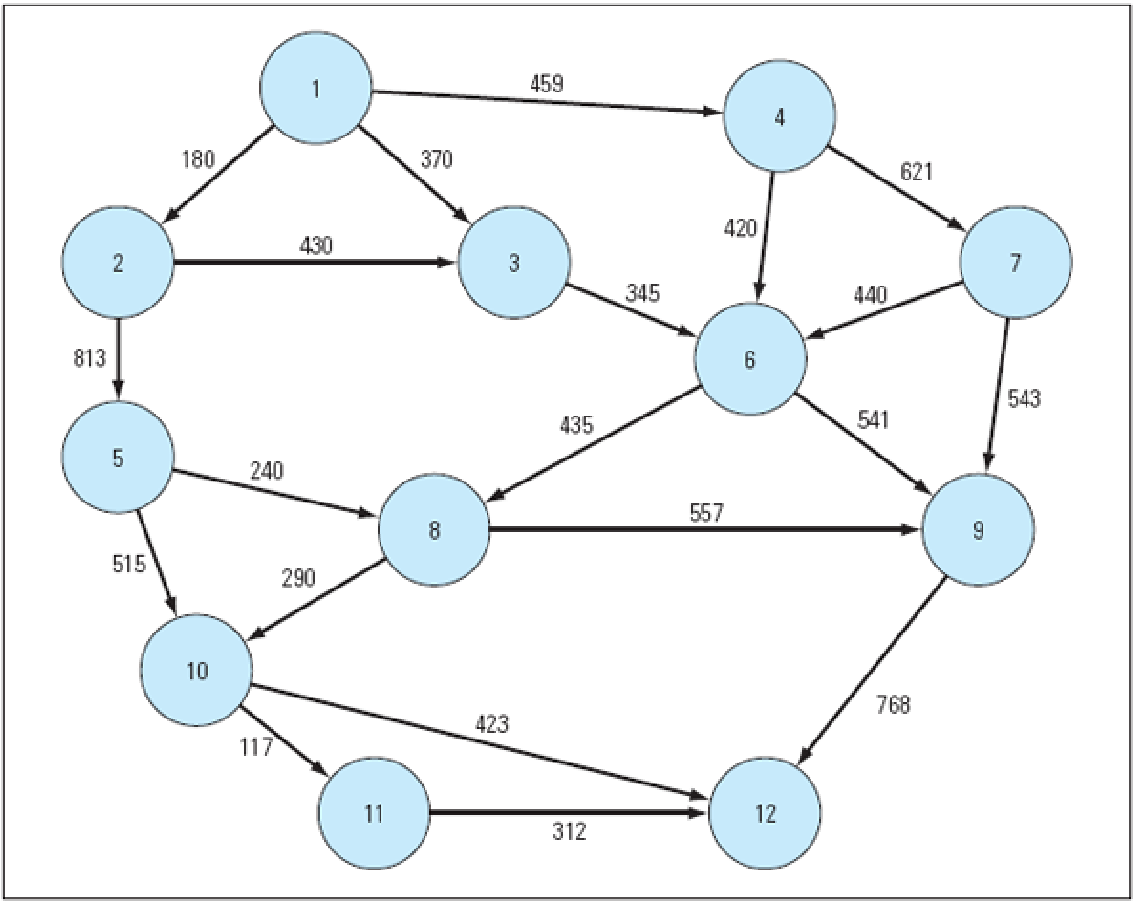
The optimal solution is:

$X_{12} = 8$, $X_{13} = 9$, $X_{14} = 7$, $X_{25} = 8$, $X_{37} = 9$, $X_{43} = 1$, $X_{46} = 6$, $X_{57} = 9$, $X_{67} = 6$.

(Alternate optima exist.)

Maximal flow = 24 tons of sewage per hour.

3. A residential moving company needs to move a family from city 1 to city 12 in Figure X where the numbers on the arcs represents the driving distance in miles between cities.



Network flow model for the moving company problem

a. Create a spreadsheet model for this problem.
b. What is the optimal solution?
c. Suppose the moving company gets paid by the mile and, as a result, wants to determine the longest path from city 1 to city 12. What is the optimal solution?
d. Now suppose travel is permissible in either direction between cities 6 and 9. Describe the optimal solution to this problem.

Solution:

a. See

Furniture Moving

Ship	From	To	Distance
0	1	2	180
1	1	3	370
0	1	4	459
0	2	3	430
0	2	5	813
1	3	6	345
0	4	6	420
0	4	7	621
0	5	8	240
0	5	10	515
1	6	8	435
0	6	9	541
0	7	6	440
0	7	9	543
0	8	9	557
1	8	10	290
0	9	12	768
0	10	11	117
1	10	12	423
0	11	12	312
		Total Distance	1,863

Nodes	Net Flow	Supply/Demand
1	-1	-1
2	0	0
3	0	0
4	0	0
5	0	0
6	0	0
7	0	0
8	0	0
9	0	0
10	0	0
11	0	0
12	1	1

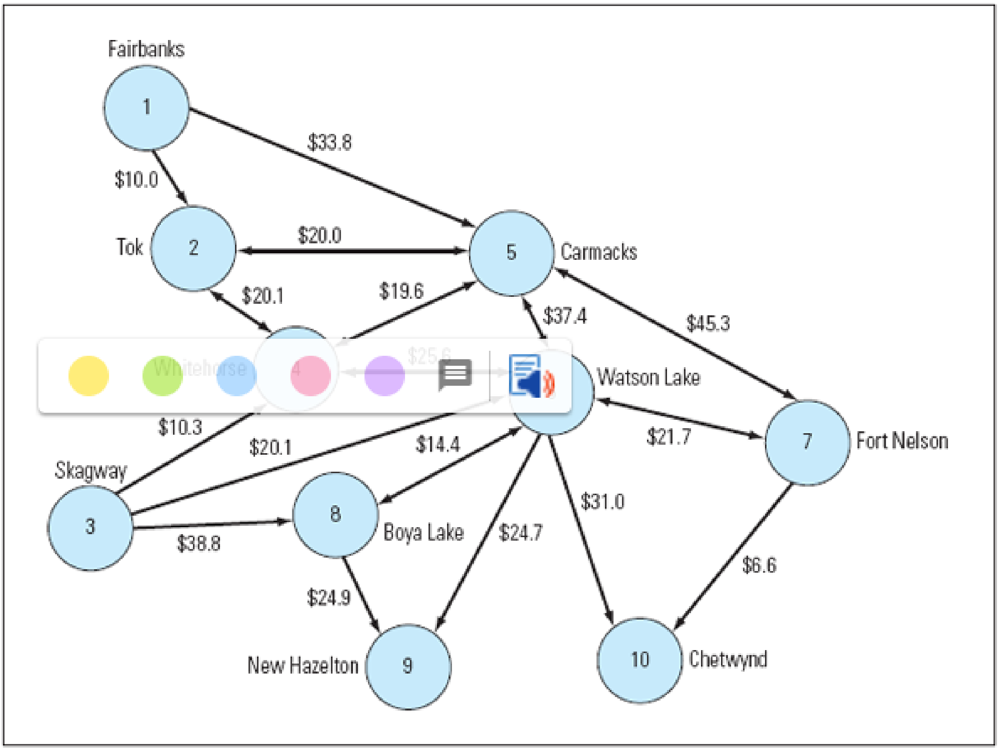
Minimize: E27
By changing: B6:B25
Subject to: H6:H17>=I6:I17
B6:B25>=0

b. 1-> 3-> 6-> 8-> 10-> 12, Total distance = 1863

c. 1-> 4-> 7-> 6-> 8-> 9-> 12, Total distance = 3280

d. For the longest route, the solution is now unbounded as a cycle between cities 6 & 9 now exists.

4. Alaskan Railroad is an independent, stand-alone railroad operation not connected to any other rail service in North America. As a result, rail shipments between Alaska and the rest of North America must be shipped by truck for thousands of miles or loaded onto ocean-going cargo vessels and transported by sea. Alaskan Railroad recently began talks with the nation of Canada about expanding its railroad lines to connect with the North American railway system. Figure X summarizes the various rail segments (and associated costs in millions of U.S. dollars) that could be built. The North American railroad system currently provides service to New Hazelton and Chetwynd. Alaskan Railroad would like to expand its railway so as to be able to reach at least one of these cities from both Skagway and Fairbanks.



Transportation options for Alaskan Railroads

a. Implement a network flow model to determine the least expensive way to connect the cities of Skagway and Fairbanks to the North American rail system.
b. What is the optimal solution?

Solution:

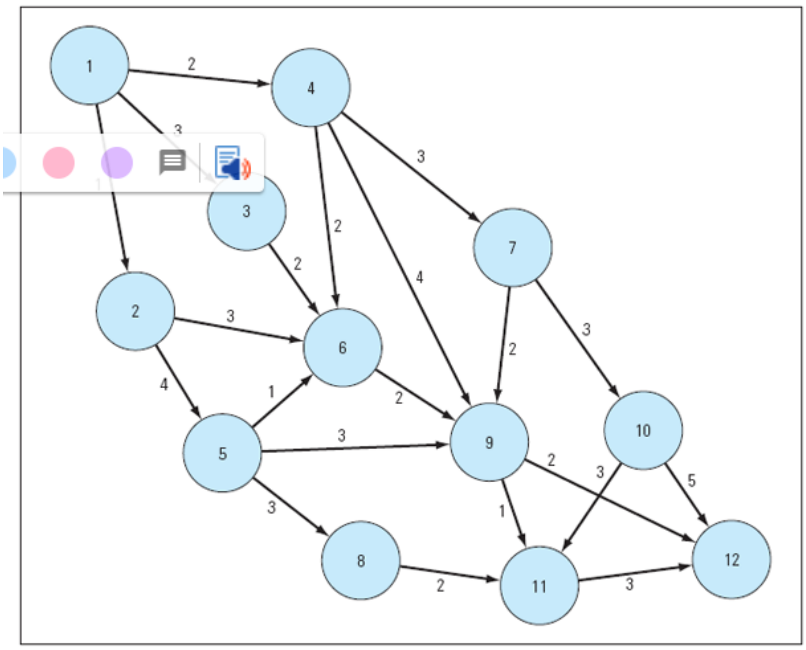
a.

	Master	Fairbanks	Skagway	From		To		Cost (000s)
0	1	1	0	1	Fairbanks	2	Tok	$10.0
0	0	0	0	1	Fairbanks	5	Carmacks	$33.8
0	1	0	1	3	Skagway	4	Whitehorse	$10.3
0	0	0	0	3	Skagway	6	Watson Lake	$20.1
0	0	0	0	3	Skagway	8	Boya Lake	$38.8
0	1	1	1	6	Watson Lake	9	New Hazelton	$24.7
0	0	0	0	6	Watson Lake	10	Chetwynd	$31.0
0	0	0	0	7	Ft Nelson	10	Chetwynd	$6.6
0	0	0	0	8	Boya Lake	9	New Hazelton	$24.9
0	1	1	1	9	New Hazelton	11	Artificial	$0.0
0	0	0	0	10	Chetwynd	11	Artificial	$0.0
1	1	1	0	2	Tok	4	Whitehorse	$20.1
1		0	0	4	Whitehorse	2	Tok	
2	0	0	0	2	Tok	5	Carmacks	$20.0
2		0	0	5	Carmacks	2	Tok	
3	0	0	0	4	Whitehorse	5	Carmacks	$19.6
3		0	0	5	Carmacks	4	Whitehorse	
4	1	1	1	4	Whitehorse	6	Watson Lake	$25.6
4		0	0	6	Watson Lake	4	Whitehorse	
5	0	0	0	5	Carmacks	6	Watson Lake	$37.4
5		0	0	6	Watson Lake	5	Carmacks	
6	0	0	0	5	Carmacks	7	Ft Nelson	$45.3
6		0	0	7	Ft Nelson	5	Carmacks	
7	0	0	0	6	Watson Lake	7	Ft Nelson	$21.7
7		0	0	7	Ft Nelson	6	Watson Lake	
8	0	0	0	6	Watson Lake	8	Boya Lake	$14.4
8		0	0	8	Boya Lake	6	Watson Lake	
						Total Cost		$91

Node		Fairbanks Net Flow	Fairbanks Supply/Demand	Skagway Net Flow	Skagway Supply/Demand
1	Fairbanks	-1	-1	0	0
2	Tok	0	0	0	0
3	Skagway	0	0	-1	-1
4	Whitehorse	0	0	0	0
5	Carmacks	0	0	0	0
6	Watson Lake	0	0	0	0
7	Ft Nelson	0	0	0	0
8	Boya Lake	0	0	0	0
9	New Hazelton	0	0	0	0
10	Chetwynd	0	0	0	0
11	Artificial	1	1	1	1

b. Minimum cost = $91 million

5. The U.S. Department of Transportation (DOT) is planning to build a new interstate to run from Detroit, Michigan, to Charleston, South Carolina. A number of different routes have been proposed and are summarized in Figure X, where node 1 represents Detroit and node 12 represents Charleston. The numbers on the arcs indicate the estimated construction costs of the various links (in millions of dollars). It is estimated that all of the routes will require approximately the same total driving time to make the trip from Detroit to Charleston. Thus, the DOT is interested in identifying the least costly alternative.



Possible routes for the interstate construction problem

a. Formulate an LP model to determine the least costly construction plan.
b. Use Solver to determine the optimal solution to this problem.

Solution:

US Dept of Transportation

Ship:	From: (Arcs)	To:	Cost
1	1	2	1
0	1	3	3
0	1	4	2
0	2	5	4
1	2	6	3
0	3	6	2
0	4	6	2
0	4	7	3
0	4	9	4
0	5	6	1
0	5	8	3
0	5	9	3
1	6	9	2
0	7	9	2
0	7	10	3
0	8	11	2
0	9	11	1
1	9	12	2
0	10	11	3
0	10	12	5
0	11	12	3
		Total:	=SUMPRODUCT

Node	Net Flow	Supply or Demand
1	=SUMIF($C$5:$C$25,F5,$A$5:$A$25)-SUMIF($B$5:$B$25,F5,$A$5:$A$25)	-1
2	=SUMIF($C$5:$C$25,F6,$A$5:$A$25)-SUMIF($B$5:$B$25,F6,$A$5:$A$25)	0
3	=SUMIF($C$5:$C$25,F7,$A$5:$A$25)-SUMIF($B$5:$B$25,F7,$A$5:$A$25)	0
4	=SUMIF($C$5:$C$25,F8,$A$5:$A$25)-SUMIF($B$5:$B$25,F8,$A$5:$A$25)	0
5	=SUMIF($C$5:$C$25,F9,$A$5:$A$25)-SUMIF($B$5:$B$25,F9,$A$5:$A$25)	0
6	=SUMIF($C$5:$C$25,F10,$A$5:$A$25)-SUMIF($B$5:$B$25,F10,$A$5:$A$25)	0
7	=SUMIF($C$5:$C$25,F11,$A$5:$A$25)-SUMIF($B$5:$B$25,F11,$A$5:$A$25)	0
8	=SUMIF($C$5:$C$25,F12,$A$5:$A$25)-SUMIF($B$5:$B$25,F12,$A$5:$A$25)	0
9	=SUMIF($C$5:$C$25,F13,$A$5:$A$25)-SUMIF($B$5:$B$25,F13,$A$5:$A$25)	0
10	=SUMIF($C$5:$C$25,F14,$A$5:$A$25)-SUMIF($B$5:$B$25,F14,$A$5:$A$25)	0
11	=SUMIF($C$5:$C$25,F15,$A$5:$A$25)-SUMIF($B$5:$B$25,F15,$A$5:$A$25)	0
12	=SUMIF($C$5:$C$25,F16,$A$5:$A$25)-SUMIF($B$5:$B$25,F16,$A$5:$A$25)	1

Minimize: D26
By Changing: A5:A25
Subject To: A5:A25>=0
G5:G16=H5:H16

US Dept of Transportation

Ship:	From: (Arcs)	To:	Cost
1	1	2	$1.0
0	1	3	$3.0
0	1	4	$2.0
0	2	5	$4.0
1	2	6	$3.0
0	3	6	$2.0
0	4	6	$2.0
0	4	7	$3.0
0	4	9	$4.0
0	5	6	$1.0
0	5	8	$3.0
0	5	9	$3.0
1	6	9	$2.0
0	7	9	$2.0
0	7	10	$3.0
0	8	11	$2.0
0	9	11	$1.0
1	9	12	$2.0
0	10	11	$3.0
0	10	12	$5.0
0	11	12	$3.0
		Total:	$8.0

Node	Net Flow	Supply or Demand
1	-1	-1
2	0	0
3	0	0
4	0	0
5	0	0
6	0	0
7	0	0
8	0	0
9	0	0
10	0	0
11	0	0
12	1	1

Minimize: D26
By Changing: A5:A25
Subject To: A5:A25>=0
G5:G16=H5:H16

The optimal solution is: X_{14}=X_{49}=$X_{9,12}$= 1, Minimum cost = $8 million